A Definition of Mathematics Problem Solving and Instructional Methodologies That Foster Student Problem-Solving Abilities

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PART I: THE PROBLEM

Introduction

Mathematics has been a central part of education for centuries. Citizens with facility in mathematics provide a country with advantages in economic growth as well as advantages in the country's ability to defend itself against its enemies. Leaders in our government have recognized weaknesses in mathematics ability in our youth, and they have invested time and money to research the problem and find solutions.

Purpose Statement

The goals of this literature review include several components: (1) definitions of problem solving, (2) identification of student challenges in problem solving, (3) explanation of three instructional methodologies for problem-solving instruction, (4) identification of the teacher's role in helping students become better problem solvers, and (5) an outline of a plan for classroom and school implementation.

PART II: THE SIGNIFICANCE OF THE PROBLEM

Introduction

Although improving, United States students have not performed well on international mathematics tests compared to some other countries (National Center for Education Statistics, 2010). Since this has been a recurring problem, many in our country have been calling for change, sometimes drastic change. Finding the real source of the problem has been a forum for debate, which certainly makes solutions more elusive. This section of the paper provides a summary of performances and concerns regarding mathematics performance of students in the United States.

Depth of Concern

The mediocre performances of United States students on international achievement tests have caused much concern about the future of the country and debate about an appropriate remedy. Results from The Third International Mathematics and Science Study (1995) ranked United States eighth graders twenty-eighth out of forty-one participating countries (TIMSS & PIRLS International Study Center, 2010). Four years later, the Trends in Mathematics and Science Study (1999) indicated little change for United States eighth graders; they were ranked nineteenth of thirty-four participating countries (National Center for Education Statistics, 2010).

The latest two Trends in International Mathematics and Science Studies indicate improvements for United States eighth graders in mathematics. In 2003 United States eighth graders were ranked fifteenth of forty-five participating countries, and in 2007 they were ranked ninth of forty-seven participating countries (National Center for Education Statistics, 2010). In addition, United States eighth graders demonstrated an

increase of sixteen score points from 1995 to 2007, the fourth most of those countries participating in both studies (National Center for Education Statistics, 2010).

Although the improvements are encouraging, some leaders in our country are still not satisfied because the issue is far too important. For instance, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (2008) begins with these compelling sentences regarding the importance of mathematical skills to the well-being of our nation:

> The eminence, safety, and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas. Leading societies have commanded mathematical skills that have brought them advantages in medicine and health, in technology and commerce, in navigation and exploration, in defense and finance, and in the ability to understand past failures and to forecast future developments. (p. xi)

The authors go on to state, “But without substantial and sustained changes to its educational system, the United States will relinquish its leadership [in mathematics] in the twenty-first century” (National Mathematics Advisory Panel, 2008, p. xi).

Average or below-average performances in mathematics in these international studies may be a threat to the future safety and prosperity of our country. What are United States teachers and leaders doing “wrong?” What can be done to change the situation? This has led to healthy debate about curriculum, pre-service and in-service teacher training, and instructional practices.

Researchers have found that improving curriculum does impact achievement. Schmidt et al. (2002) studied the curricular content of the six 1995 Third International

Mathematics and Science Study (TIMSS) top-performing countries and found much similarity. United States schools participating in the study had curriculum similar to each other but different from the curriculum of the top-performing countries. Researchers opined that curricular differences explained differences in test scores. The top-performing countries had a more focused curriculum with fewer topics, less repetition, more coherence from topic to topic, and increased difficulty. The researchers called a curriculum with these aspects a "quality" curriculum.

Hook, Bishop, and Hook (2007) studied data from the California Department of Education from 1998 to 2002. Elementary and middle school student achievement in school districts that adopted the new (1998) math curriculum in California (very similar to the "quality" curriculum above) was compared with those that did not. Students in schools with the new (1998) curriculum significantly outperformed those using the old (1991) curriculum. A "quality" curriculum positively impacted student achievement in both low socio-economic districts as well as in one high socio-economic district.

Although a quality curriculum seems to improve student mathematics achievement, there are other factors to consider. Ma (1999) studied the differences in mathematical knowledge between Chinese and United States teachers. She found Chinese teachers had far greater depth of conceptual understanding. This knowledge gap between teachers parallels the gap in student performance in the two countries. *No Common Denominator: The Preparation of Elementary Teachers in Mathematics by America's Education Schools* (2008) indicates several failures in teacher mathematics training programs in 77 colleges of education across the country. These deficits included the following: inadequate or absent requirements for program entry and exit, poor textbooks,

and a lack of demanding content. The researchers recommend three courses in elementary mathematics content and one course teaching mathematics methods. They also recommend that teachers be required to take a mathematics content test to demonstrate sufficient content knowledge.

What do in-service teachers need to know about teaching elementary and middle school mathematics? As an instructional leader of a small elementary school, this researcher feels compelled to learn and share effective teaching methodologies for mathematics. The National Mathematics Advisory Panel (2008) found that teacher content knowledge affects student performance, but research did not identify specific mathematical knowledge and instructional skills needed for effective instruction.

Research indicates a strong emphasis on problem solving (National Council of Teachers of Mathematics, 2000; Hiebert et al., 2003; Wilson & Blank, 1999; Ben-Hur, 2006; Davis, Maher, & Noddings, 1990; Checkley, 2006). In *Principles and Standards of School Mathematics* (NCTM, 2000) educators are urged to teach mathematics through problem solving. What does that mean? Does research support this kind of instruction? Are certain ways of teaching problem solving more effective than others?

Conclusion

The problem of poor student performance on international tests is indeed a complex one. Many factors may influence positive change: curriculum, pre-service teacher education, in-service teacher training, and classroom instruction. Which is more significant? The answer likely depends on the individual's perspective, but research clearly indicates that change is necessary for the stability and safety of our country (National Mathematics Advisory Panel, 2008).

PART III: REVIEW OF LITERATURE

Introduction

As an instructional leader, this researcher is most interested in best practices for in-service teachers. Since problem solving is the heart of mathematics, the literature review begins with a look at varied definitions used for "problem solving." Next, three research-based instructional methodologies will be described. Finally, the teacher's role in problem solving instruction will be considered.

Varied Meanings Problem Solving

Problems in mathematics may range from simple exercises designed to produce automaticity of basic facts to elaborate problems set in rich contexts. Checkley (2006) quotes Barbara Reys, a mathematics professor at the University of Missouri, as saying, "Many people equate problem solving with using an algorithm to solve an equation" (p. 81). Researchers and national organizations would refute this definition (National Council of Supervisors of Mathematics, 1989; National Council of Teachers of Mathematics, 2000; Ben-Hur, 2006; and Kenney, 2005).

The National Council of Supervisors of Mathematics (NCSM) defines problem solving as "the process of applying previously acquired knowledge to new and unfamiliar situations" (NCSM, 1989, p. 471). This definition focuses the attention on using skills or knowledge to problem situations, which is more than solving an equation.

The National Council of Teachers of Mathematics (NCTM) says "problem solving means engaging in a task for which the solution method is not known in advance" (NCTM, 2000, p. 52). As students acquire skills and knowledge about mathematics, the sophistication of this type of problem will increase. In order to solve a problem, one must

consider which factors are important to finding a solution, and then one must determine a pathway to a solution.

Ben-Hur (2006) says, “Problem solving requires analysis, heuristics, and reasoning toward self-defined goals” (p. 71). As a person encounters a problem, he should contemplate a few questions to help him dig deeply for details necessary to solve the problem. The following questions are examples of good questions to consider when approaching a problem: What information does the reader know from the problem? What other information is necessary to solve the problem? How are pieces of information connected or related? What is the goal of the problem? What is a way to reach that goal?

Kenney (2005) describes problem solving as a process that involves such actions as modeling, formulating, transforming, manipulating, inferring, and communicating. Problem solving involves taking the information in the problem and translating it into another medium in order to better identify solution strategies. For instance, a problem-solver could take an equation and make a graph, or a student could take the words of a problem and make a diagram.

Many people may misunderstand problem solving as the application of an algorithm (Checkley, 2006). However, problem solving may be better defined as a much more complex and rigorous process of understanding the problem situation, making a plan to find a solution, solving the problem, verifying the solution, and considering alternative solutions or pathways to a solution.

Methodologies for Teaching Problem Solving Skills

One way to teach problem solving is to help students identify key words in the problem and translate them into mathematical ideas or symbols. Words and phrases such

as *times*, *plus*, *subtract*, *increased*, *quotient*, *multiply*, *less than*, *half*, *triple*, and *equal* all relate to mathematical concepts. Using a key word strategy, students find these words in a problem and use them to determine a solution path. As an example, consider this problem: "Two flags are similar. One flag is three times as long as the other flag. The length of the smaller flag is 8 in. What is the length of the larger flag?" (Xin, 2008, p. 535). The word "times" suggests multiplication, so the teacher would help students identify this word and its meaning to determine a solution path: 8×3.

Teaching students to solve problems using key words has been done for many years (Xin, 2008), but some believe it undermines real problem solving (Xin, 2008; Ben-Hur, 2006). Using mathematics to solve problems involves uncertainty (Ben-Hur, 2006), and students who are taught to mechanically change words into symbols are prone to mistakes when the language is inconsistent (Xin, 2007). A slight rewording of the previous problem illustrates this inconsistent language: "Two flags are similar. One flag is three times as long as the other flag. The length of the larger flag is 8 in. What is the length of the smaller flag?" Using a key words approach, many students identify the word "times" and multiply eight and three. This typical error is understandable and stems from a misunderstanding of the problem and the relationship between its parts. Ben-Hur (2006) warns against teaching key words out of context, saying this can lead students to missing the underlying mathematical ideas the words represent.

Understanding the language of mathematics is indeed critical to successful problem solving (Kenney, 2005; Pape, 2004). The language of mathematics contains many confusing terms: *base*, *radical*, *pi*, *prime*, *power*, *variable*, *sine*, *sign*, *right angle*, *compute*, *dividend*, *factor*, *mean*, etc. Some words have specialized meanings when used

in a mathematical context, and these meanings may be similar to or different from their meanings in everyday language. For instance, the word *power* in everyday language refers to strength, but in mathematics it refers to the exponent on a number or variable. Even the words *of* and *off* (only one letter difference) can be very confusing to students: thirty percent of a number versus thirty percent off the regular price. Consider the use of the word *difference* in mathematics compared to its use in everyday language. What does the word *mean* mean? Add up mathematics terms associated with types of numbers (integer, whole, composite, prime, rational, irrational, etc.), shapes (square, rectangle, triangle, trapezoid, triangular pyramid, hexagonal prism, scalene quadrilateral, etc.), and graphs or plots (bar, broken-line, histogram, circle, box-and-whisker, stem-and-leaf, etc.) and it is not difficult to conclude that the language of mathematics requires a great deal of specialized terminology. Misunderstanding or misinterpreting any of these words will undermine successful problem solving, so teachers must carefully and systematically build students' mathematical vocabulary through many experiences. There is much more to successful problem solving than simply finding and translating key words into mathematical sentences or defining mathematical terms.

Another method to help students become better problem solvers is to teach specific strategies for solving problems. Many sources of problem solving strategies are available (Problem Solving in Mathematics, n.d.; Word Problem Solving Strategies, n.d.; Liebsch, 2008). Some of the most common are: make a model or diagram, make a table or list, look for patterns, use an equation or formula, consider a simpler case, and guess and check.

In teaching students strategies, teachers may approach the method in different ways. Some may model the strategy on a given problem, and then ask students to use the strategy on similar problems (Rickard, 2005). Another way to teach strategies involves teacher modeling, but then a variety of problems are posed in which the strategy might or might not work. A given problem may be solved using more than one strategy.

As an example, consider the following question: “One square has a perimeter of 40 inches. A second square has a perimeter of 36 inches. What is the positive difference in the areas of the two squares?” (Liebsch, 2008, p. 23). To solve this problem, students may choose the strategy “Use an Equation or Formula.” The perimeter of a square can be expressed by the formula $P = 4S$. Using this formula, students could determine the side lengths for each of the squares as 10 inches and 9 inches. The formula for the area of a square is $A = S^2$. Thus, the area of the larger square is 100 square inches, and the area of the smaller square is 81 square inches. The difference between the areas of the two squares, then, is found by subtracting the smaller area from the larger area. Another student may solve this problem choosing the strategy, “Make a Model or Diagram.” This student may use graph paper to draw one square inside the other and count the difference.

Research supports the teaching of strategies to improve problem solving abilities (Rickard, 2005; Jitendra, DiPipi, & Perron-Jones, 2002; Higgins, 1997; Mastromatteo, 1994), but research does not support the teaching of strategies to improve overall mathematics achievement. How strategies are taught and used by students significantly impacts the usefulness of this approach. Teachers want to avoid encouraging students to use problem-solving strategies as an algorithm or simply applying them mechanically to problem situations (Schoenfeld, 1988).

A third method for teaching problem solving is called schema-based instruction. Schema-based instruction means using models to represent the schematic structure of a given problem. Nesher & Hershkovitz (1994) found that students' difficulties with problem solving could be predicted by the complexity of the underlying schemata. The schematic relationship between parts might be simple or complex (Ben-Hur, 2006). Predictably, students tend to have more difficulty with problems that have more complex schematic structures. Students also have more difficulty with problems that involve non-commutative operations (Carpenter, Fennema, Franke, Empson, & Levy, 1999; Nesher & Hershkovitz, 1994). Xin (2008) found that schema acquisition is an important part of successfully solving problems.

The model in schema-based instruction is designed to help students recognize the relationship between values in a given problem and make a reasonable plan to solve the problem. Xin (2008) found that students with learning disabilities improved after direct, one-on-one schema-based instruction. The students were also fairly successful at generalizing the skills to new problems and maintaining those skills weeks after instruction.

Ben-Hur (2006) identifies four simple and three complex schemata. Simple schemata are applied to one-step solutions. An example of a problem requiring simple schemata would be one like this: "Juan has twenty-two baseball cards. Rob has sixteen baseball cards. How many cards do they have altogether?" Here is a possible model for this type of problem: $\Delta = \Diamond + \square$. With known quantities placed in the model, it looks like this: $\Delta = 22 + 16$. The problem could easily be changed to leave the total quantity unknown, but this would not change the schematic model used to solve the problem. If

the problem read, “Juan has twenty-two baseball cards. Rob also has baseball cards. Together Rob and Juan have thirty-eight baseball cards. How many baseball cards does Rob have?” The model above would still be used. It would look like this with known quantities placed in the model: 38 = 22 + □.

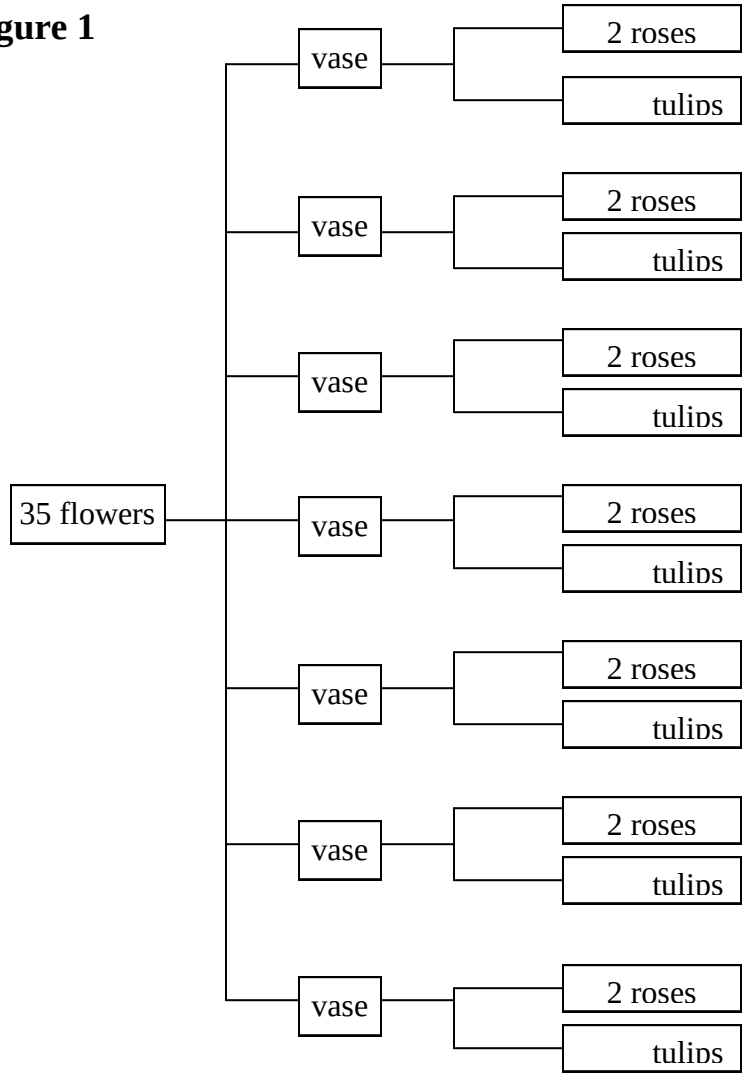
A very similar schematic structure can also be used for problems involving multiplication. For instance, consider the flag problem referred to on page 12. A schematic model for the problem could be Δ = ◊ × □. When the length of the smaller flag is known, the structure is Δ = 8 × 3. When the length of the larger flag is known, the structure is 8 = ◊ × 3. This structure can also be used when there is a partial relation. For instance, the flag problem could be reworded: “Two flags are similar. One flag is one-third as long as the other flag. The length of the smaller flag is 8 in. What is the length of the larger flag?” The model would be 8 = 1/3 × □.

Complex schemata have different models and require two or more steps to solve the problem. Ben-Hur (2006) identifies three types of complex schemata: hierarchical, sharing whole, and sharing part. The hierarchical scheme is the easiest for most students, while the sharing part scheme is the most difficult. An example of a problem involving a hierarchical schematic structure is, “A total of 35 flowers are distributed evenly among seven vases. Each vase contains two roses. The rest are tulips. How many tulips are in each vase?” (Ben-Hur, 2006. p. 91). Figure 1 shows a hierarchical schematic structure for this problem.

The sharing whole schema includes two groups or structures that share one whole. An example problem would be, “There are twenty boys and twelve girls in the camp. They are equally divided into four groups. How many children are there in each group?”

(Ben-Hur, 2006, p. 91). Figure 2 shows a sharing whole schematic structure for this problem.

Figure 1



The sharing part schema includes two groups or structures that share one part. An example problem would be, “At the party there were twenty children, twelve of whom were boys. The forty flowers that were left from the party were distributed equally among the girls. How many flowers did each girl get?” (Ben-Hur, 2006, p. 91). Figure 3 shows a

sharing part structure for this problem. Notice that the row of boxes on the right is not complete. A calculation on the left side of the schemata will determine the number of boxes needed on the right.

Figure 2

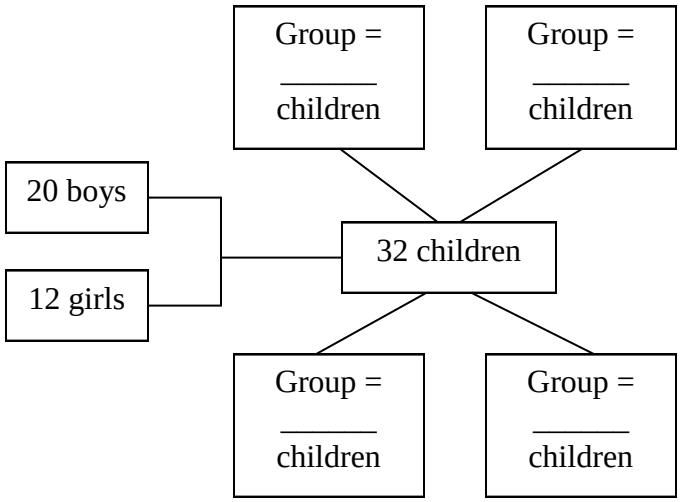
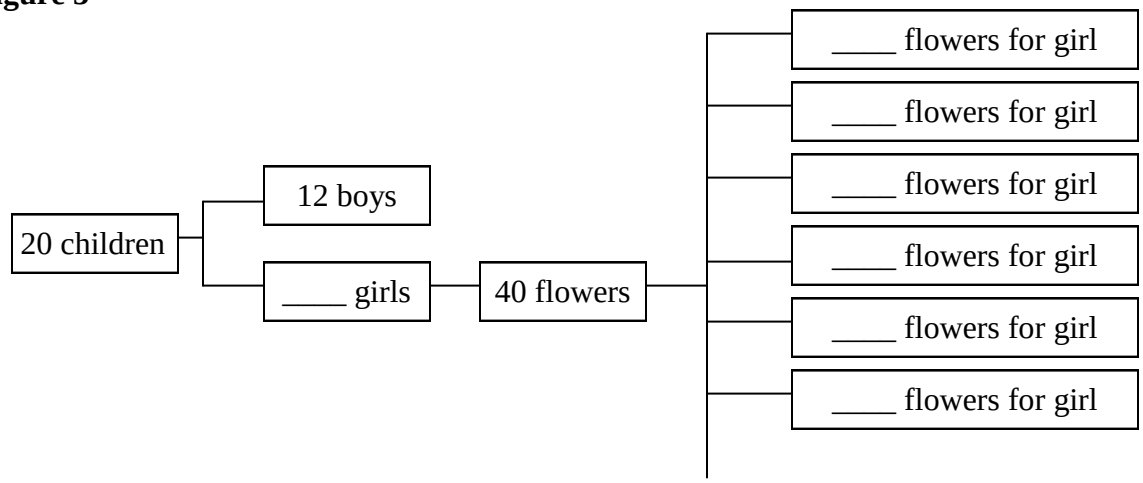


Figure 3



The schematic models help students visualize relationships between problem elements. To be successful, students will need clear instruction on the meanings and uses of the schematic models, and then practice matching schematic models with appropriate problems. Xin (2008) used scripted lessons for one-on-one instruction with learning disabled students. While the number of strategies a student might use to solve a problem could be almost endless, the number of schematic models may be relatively small. Combinations of simple and complex schematic models are effective for solving many types of problems (Ben-Hur, 2006). Using schematic models drives students to consider the relationships between quantities in a problem before choosing a model to use.

The Teacher's Role in Fostering Problem-Solving Skills

To better understand what a teacher should do to foster problem-solving skills in students, it is helpful to understand typical reasons for student errors. Many student errors stem from misconceptions, especially preconceptions. Preconceptions may be either undergeneralizations or overgeneralizations.

When students are beginning to learn about a concept, they often undergeneralize the meaning. For instance, a student may fail to recognize 3:4 as equivalent to 3/4. Representing a number with a colon separating the two parts would likely be learned in conjunction with ratios, while the fraction bar might be studied with fractions or division. While learning about the equal sign (=), students tend to believe it means to perform certain operations to find the solution; students fail to generalize the idea of equality when it comes to algebraic equations.

Overgeneralizations occur when students believe that multiplying two numbers always produces a larger number, or when they misapply the concept of "borrowing" or

“regrouping” with multi-digit subtraction. Students overgeneralize the rule for addition and subtraction of fractions when they find common denominators for problems involving multiplication. Many teachers are drawn, and students encourage this, to provide students with “rules” to follow instead of helping them better understand the concept or why the “rule” works. As students get older, the inundation of “rules” for doing math may overload their circuits and undermine their understanding and ability to use math meaningfully.

What is a better way for teachers to respond to these misconceptions? Ben-Hur (2006) offers six principles to guide instruction: reciprocity, flexibility, alternative mental representations, metacognitive awareness, appropriate communication, and constructive interaction among learners. In order to adjust or change an individual’s misconceptions regarding mathematic concepts, active participation in dialogue based on respecting each other’s opportunity to share ideas produces the reciprocity necessary for the change to occur. Judgmental responses, however, will produce defense mechanisms in participants and hinder learning. Teachers who base lesson content on the particular needs of students demonstrate the flexibility needed to change incorrect misconceptions. These teachers may use different activities, spend more time on a topic, or adjust objectives. Presenting concepts in a variety of ways, teachers provide students with frameworks that allow them to consider concepts beyond what can physically be manipulated. These representations may include oral, written, graphic, symbolic, or pictorial forms. Most students who have false preconceptions about mathematics do not quickly change their ideas when presented with counter-examples or conflicting new evidence. Teachers often need to lead students in mental reconstruction through probing questions during problem solving. The

questions aim to direct students to their thinking processes. Graphic organizers also aid in reconfiguring concepts. Mathematics has a rich language of its own, and student understanding of the various terms associated with different concepts presents a definite challenge to clear understanding. Teachers cognizant of students' developing mathematics vocabularies will find ways to clearly communicate ideas and interpret student responses. Dialogue among students can change misconceptions when it fosters higher order thinking as well as personal responsibility and group responsibility. Problems discussed in groups should encourage students to debate and clarify the problem so that each group member understands the question, at least one correct solution path, and reasons that path works for the given problem.

In summary, classrooms where students listen to each other and respect each other's ideas will foster dialogue for successful problem solving. When teachers adjust instruction based on feedback from classroom dialogue, students have a chance to correct misconceptions. If students learn to create different ways to represent mathematical concepts, they gain a deeper understanding of the concept. Using questionnaires with students, modeling meta-cognition, and posing questions to students can help them develop thinking skills necessary for persistent and successful problem solving. There are also specific programs designed to help foster meta-cognition (deBono, 1985; Feuerstein, 1980; Lipman, 1984; Palincsar & Brown, 1984).

There are other reasons why students fail to solve problems successfully. Some students lack the ability to create an appropriate image fitting for the problem's context (Novak, 1990). Teachers who insist on seeing student work will gain insights into students' imaging abilities. Some students simply cannot maintain the original problem

while processing parts of it (Campbell, Collis, & Watson, 1995). Other students lack logical thinking skills and the ability to apply them to problem situations (Koontz & Berch, 1996). Formative assessment, assessment performed during teaching designed to guide instruction, will help teachers in handling these challenges, but a basic framework will help all students attack problems with a purpose.

Much current research in problem solving is based on the process described by George Pólya. His writings have been used as a basic framework for much research in mathematics problem solving for decades (Ben-Hur, 2006). Although textbooks may present Pólya's work as a set of steps to use for solving problems in a linear fashion, the process is actually flexible and fluid. The four components of Pólya's process include understanding the problem, making a plan, carrying out the plan, and looking back (Pólya, 1945).

What does it mean to understand the problem? It is more than simply reading the words. Understanding the problem involves identifying necessary and unnecessary information, determining if other information is needed, stating what is known and what is unknown in the problem, recognizing when calculations must be made prior to making other calculations, and rephrasing the problem when it helps clarify the goal. Teachers should guide students, through discussion, to consider these ideas prior to attempting to solve a problem. Teachers should lead students to study the relationship of the parts of the problem.

Research indicates that students often fail to realize the importance of understanding the problem and also lack the ability to do so (Ben-Hur, 2006). One way to help students develop these skills is to pose a problem situation without a question; then

ask students to suggest possible questions that fit the situation. It is important that teachers foster an environment where students feel comfortable asking questions; it is a risk students need to take to clarify a problem. If students fail to reflect upon and clarify components of a problem prior to attempting to solve the problem, they generally proceed aimlessly.

Teacher questions are essential in helping students understand the problem. Clement and Bernhard (2005) provide questions for teachers to pose to help students understand the situation:

> What quantities are involved in this situation? What quantities am I trying to find? Which quantities are critical to the problem at hand? Are any of these quantities related to each other and if so how? Do I know the values of any of the quantities and if so which ones? (p. 365)

The questions help students seek known and unknown values and the relationships between them.

What does it mean to make a plan? Once a student understands the problem, he needs to decide how he might go about solving the problem. Students who have been taught different problem-solving strategies might select one of these. Those taught to use a schema-based model would determine which model or models apply to the given problem. Considering ways to solve a problem involves a great deal of metacognition. (Metacognition is an awareness of one's thought processes. Stated simply, metacognition is thinking about thinking.) Teachers can model this metacognition by telling students the thought processes he is going through when he is making a plan to solve a problem. Classroom discussion should also allow students the opportunity to share their thoughts

while making a plan and responding to the suggestions of classmates. Creating a visual representation can lead to acceptance or rejection of different plans.

What does it mean to carry out the plan? After a plan has been made, students follow it. Again, metacognition should always be happening. Ben-Hur (2006) provides an excellent list of possible questions for guiding metacognition while solving a problem:

> How am I doing? Am I on the right track? How should I proceed? What should I do to keep track of what I have already done? Should I move in another direction (revise the plan)? What do I need to do if I don't understand? How far am I into the process? (p. 104)

Students who lack meta-cognitive skills end up on "wild goose chases" (Schoenfeld, 1987). It is important for teachers to help students develop self-awareness and self-regulation by asking them to clarify what they mean, compare solutions or strategies to find a solution, and interpret the meaning of the solution.

What does it mean to look back? Pólya (1945) considers looking back as the most critical step in the process. After a solution is reached, students should check this result to make sure it is fitting for the problem. Students should also consider the choices and strategies they used to solve the problem and the consequences of those choices, consider or create other problems that could be solved the same way, and suggest changes to the problem and how those changes would affect the solution. Teachers should help students make generalizations about rules and concepts that will help them with further problem solving.

Conclusion

There are challenges for many in-service teachers to overcome to be effective mathematics teachers. As they read about problem solving, they may be confused by the different ways the term is used in the literature. Many teachers have not had sufficient coursework to prepare them for effective mathematics teaching. As a result, these teachers may have much to learn about mathematics as a subject as well as best-practices for classroom instruction. Teachers should be knowledgeable about the language of mathematics, but they should resist encouraging a formulaic approach of solving problems by translating words into mathematic sentences. There are many valuable tools available to help teachers learn problem solving strategies and ways to guide students in the proper use of these strategies. Teachers should avoid encouraging students to apply the strategies mechanically. Schema-based instruction involves using schematic structures to help students visualize the relationships between problem elements. Teachers can use simple or complex schemata to help students build a framework that will help them solve problems with similar structures. Teachers need to provide students with experiences using the schematic structures and applying them to a variety of problem situations. The teacher's role in problem solving instruction is much different from traditional teaching methods, like direct instruction. Consequently in-service teachers may need substantial support when attempting a "guide on the side" approach to instruction.

PART IV: IMPLEMENTATION

Introduction

The problem is serious, and the task is daunting. The solution lies in taking one step at a time. This final section presents a plan for producing positive change in the instruction and learning of mathematics in this researcher's classroom and school.

Plans for Classroom

In his classroom of sixth through eighth grade students, this researcher intends to affect positive change regarding attitudes toward problem solving as well as skillfully attacking and finding solutions to the problems. To accomplish this, this researcher will discuss assessment tools to measure progress toward these goals and teaching plans for producing the desired outcomes.

An attitude survey (Appendix A) and a problem solving portfolio will be used to assess students' attitudes toward mathematics and problem solving and their skills in doing so. The attitude survey will incorporate a Likert scale for many of the statements, and it will be administered both at the beginning of the school year and near the end of the school year. Students will use a specialized form (Appendix B) for showing work and reasoning in their problem solving portfolio. The researcher will demonstrate how to solve a problem using the form for each of the first three problems and periodically thereafter as deemed necessary to improve student work. Copies of the form will be readily available in the classroom.

The Everyday Mathematics (2007) curriculum, which is in place St. John's Evangelical Lutheran School, is designed to teach mathematics through problem solving. As a result, problem solving experiences occur almost daily. In addition to these

experiences, weekly problems will be presented to the class as an additional challenge. These problems will come from a variety of sources: Meet Math competition sheets, Math Counts Web site, *Mathematics Teaching in the Middle School* magazine, and additional problems from the mathematics curriculum. The additional problems will be presented at the beginning of the week, and culminating discussions will occur at the end of the week. As often as possible, the problems selected will be used to address particular misconceptions held by students in the class. The problems will also be used to develop deeper understanding of current math topics, review past topics, preview future topics, and link to other areas of the curriculum.

The first approach to problem solving that will be presented to the class will be Pólya's method. Each student will be given a copy of the main ideas or steps of the method as well as guiding questions. A poster version will also be displayed in the classroom. To help students develop their ability to understand the problem, early problems will be initially presented without a question. Students will be led to identify known and unknown information, connections between values in the problems, potential resources for further information, and any other information pertinent to the problems. Students will also be asked for potential questions from the situation and information provided in the problems. After choosing a question, students then will discuss potential plans for solving the problem, always providing reasons for their choices. Further discussion will follow after possible solutions are reached.

Although Pólya's method will be the overarching method encouraged to attack problems, other problem solving strategies will be presented as well. Each of the six strategies listed earlier in this literature review (make a model or diagram, make a table or

list, look for patterns, use an equation or formula, consider a simpler case, and guess a check) will be presented using an example problem. As new strategies are presented, students will be asked if the current example problem could be solved using a different strategy. They will then be asked to explain why they believe it can or cannot. If they believe it can be solved with another strategy, they will be asked to do so. When multiple strategies may be used to solve a problem, students will be asked which strategy they prefer for the particular problem and why they prefer it. The goal will be to get students to find strategies that are both efficient and effective.

Middle school students often appreciate choice and active involvement. After ten weeks of problem solving instruction, students will be encouraged to find or create their own problems to challenge their classmates. Individually and in small groups, students will prepare a problem and a solution method for this researcher. After checking for potential errors, these problems will be presented to the class as weekly problems. In order to keep all students working on problem solving, two problems will be presented each week. Students may choose one of the problems to solve, but they may not choose their own problem.

Plans for School

Improving mathematics instruction in the school begins with small steps. In a summer in-service, teachers will discuss the value of assessing students' attitudes toward mathematics and problem solving and as well as their problem solving skills. The attitude survey (Appendix A) used in this researcher's classroom will be presented to the other teachers for discussion. The staff will decide to adopt this survey, make changes to the survey, or create grade level surveys. The portfolio form used in this researcher's

classroom will be presented to the teachers for discussion as well. Because young students do not have strong writing skills, an alternative assessment will need to be devised for these children. Some assessments for younger children incorporate sad faces, smile faces, and faces with straight mouths instead of words. The assessment would likely have fewer statements because an adult would need to read the statements to each child. Some statements may need simpler wording to match the comprehension level of the students. This researcher intends to enlist the grade-level experience of the other teachers to help create the alternative survey.

Next, the teachers will discuss methodology for teaching problem solving skills. To begin, teachers will discuss Pólya's method and apply it to problems at various grade levels. This researcher will provide his plan for problem solving instruction, and teachers will be asked to create a year-long plan of their own. These plans will be discussed at later in-service sessions.

Each year our school has a teacher workday about midway through the first quarter. At this workday, this researcher will present the problem solving strategies listed earlier in this literature review. Teachers will evaluate the appropriateness of each strategy for their grade level. Teachers will also provide at this meeting a simple update on problem solving in their classrooms: Have they been following the plan they created? How have students responded? Each teacher will provide a summary of their attitude survey. This summary will include the number of responses in each category to each statement and the teacher's reaction to these responses.

During first-quarter faculty meetings, this researcher will share two common misconceptions: incorrect preconceptions (there is no number lower than zero, fractions

always represent parts of a whole, and measurement starts at one instead of zero) and overgeneralizations (products are always bigger than their factors, when someone is adding he always lines up the numbers on the right side, and power represents multiplication). Teachers will then be asked to identify other misconceptions frequently held by students in their grade level. This researcher will also visit each classroom once during each quarter to observe a mathematics class. One of the observation goals will be to identify any misconceptions held by students. Since the faculty of St. John's Evangelical Lutheran School is comprised of teachers with years of experience ranging from twelve to thirty-six, this researcher feels confident that teachers will be able to identify other common student misconceptions at their particular level.

Part of our end-of-quarter faculty meeting will be used to address student misconceptions. Using the six principles from Ben-Hur (2006), which are highlighted earlier in this literature review, the faculty will discuss possible ways to correct these student misconceptions. Teachers will attempt the solutions devised and report their findings at our December faculty meeting. Each successive faculty meeting will allow for updating and generating new ideas for these misconceptions. Teachers will also present updates on student problem solving skills at each quarterly meeting. The quarterly results will also be shared with the Lutheran Elementary School Committee, the body responsible for school oversight.

Finally, teachers will read and discuss current literature on mathematics instruction monthly. *Mathematics Teaching in the Middle School*, *Teaching Children Mathematics*, and other sources will be made available to the teachers. At our monthly faculty meetings, each teacher will take a turn sharing something she has learned about

teaching mathematics at her grade level. This researcher will, from time to time, provide research related to current topics.

Conclusion

Lutheran elementary school teachers often teach many different subjects: religion, grammar, reading, writing, spelling, social studies, science, music, art, physical education, mathematics, and even more. How can they reasonably be expected to be expert teachers in all of these areas? Our schools must maintain a focus on training and instructing in God's Word, for failure to do so has eternal consequences. However, it is possible to maintain our focus while improving our instruction in other areas.

Part of the mission statement of St. John's Evangelical Lutheran School states that the school will maintain a high standard of education. Mathematics instruction is a critical component of our education. The need for quality mathematics instruction is clear, for the student, the school, and the country. Although Lutheran elementary school teachers are busy, they will find time to grow in their abilities in humble service to those the Lord has placed in their care. This includes talking with fellow teachers about mathematics instruction, creating classroom environments conducive to the development of problem-solving skills, and continuing to learn methods and strategies that will better enable us to help students learn. May the Lord direct our paths as we strive to help His children learn mathematical problem solving skills and attitudes.

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Appendix A

Student Mathematics Attitude Survey

As your teacher, I am interested in your views of mathematics. Your answers will help me better be able to teach you.

This survey is not something that is graded, and I will not share your personal answers with anyone, unless you give me special permission. I do intend to share overall class answers with other teachers and with the Lutheran Elementary School Committee to help us plan for the very best instruction.

Please place an "X" in the box that best describes how you feel about each statement.

Statement	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1. I enjoy math.					
2. I am good at math.					
3. I usually understand what we are doing in math class.					
4. Doing math makes me nervous or upset.					
5. Math is basically memorizing facts and steps to solve problems.					
6. In math a person can be creative and discover things by herself.					
7. Good math students can solve a problem in 2 minutes or less.					
8. If I haven't solved a problem in 2 minutes, I stop.					
9. I best am able to solve a problem on my own.					
10. I best am able to solve a problem when I work with others.					
11. Math is useful in everyday life.					
12. Math will be important for me as an adult.					
13. I find math classes not challenging enough for me.					
14. I want to learn more about math.					
15. I enjoy challenging math problems.					

Appendix B

Problem Solving Form for Portfolio

Problem	
Written description/Reasons for work	**Calculations**
Solution	